SAINT JOSEPH COLLEGE
CONNECTICUT

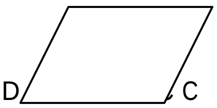
Center for Academic Excellence

Area and Perimeter

There are many formulas for finding the area and perimeter of common geometric figures. The figures in question are two-dimensional figures; i.e., in some sense they have length and width.
A line, in contrast, has only length (one dimension).

First we will consider a rectangle: a four-sided figure with four right angles and consequently a member of the class of 4-sided figures called parallelograms. Recall the definition of a parallelogram: a 4-sided figure with opposite sides parallel and equal.

A B



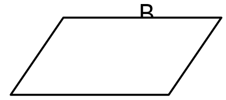
D C

AB || DC AD || BC AB = DC AD = BC

Glossary: || means parallel or "is parallel to" AB means "the measure (length) of AB"

Of course, sometimes we are a bit casual and say simply AB= DC and mean the same thing, their lengths are equal. The marks on the figure indicate equality.
The mark, ' , on AB and DC means they are equal in length;
Similarly, " , indicates that AD = BC.

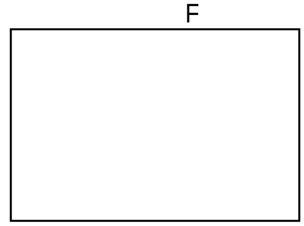
A B



D C

We started by remarking that there are many formulas for finding the area (we'll consider perimeter later) of two-dimensional (also called plane) figures.

E F



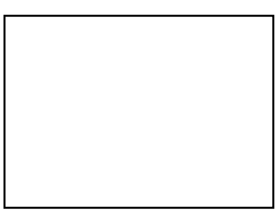
w=2

H l=3 G

Look at the rectangle with length of 3 units (inches, feet, centimeters, etc.) and width of 2. The square (a rectangle with all 4 sides equal), Q is one unit (let's say 1 inch) on a side and thus is 1 square inch. You can count the number of square inches in the figure EFGH. It is clearly 6. When you look at its dimensions, length, l = 3 inches, width, w = 2 inches, can you see how to find the area in a quicker manner than counting the square inches? Of course, you multiply the length by the width; in symbolic notation: A=lw. In this case, A = 3(2) = 6 sq. in. Any letters may be used to represent the dimensions; for example, we might use b and h, which conveniently are the initials of base and height which are commonly used terms in geometry.

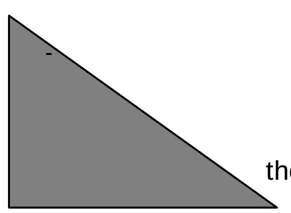
The preceding discussion was intended to suggest that you do not have to memorize the formula for the area of a rectangle; rather, with a quick sketch, you can figure it out for yourself. Now, let's use the formula, A = bh, to derive some other formulas. Take a rectangular piece of paper, and cut it (or imagine cutting it) as shown:

h



b

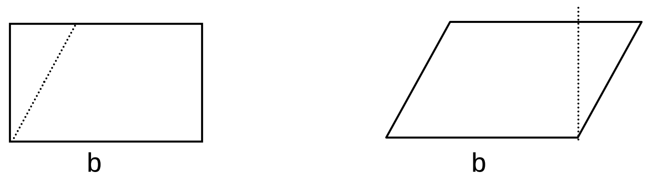
h



b

Now you have a triangle (actually two). What is the area of the shaded one? Clearly we cut the paper in half so if the entire (rectangular) sheet had an area, A=bh, what is the area of the triangle? $A = \frac{1}{2}bh$ Do you have to memorize it when it's that easy to figure out?

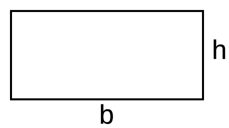
Now take another rectangular sheet and cut along the dotted line (in imagination if you prefer). The result, if you move the little triangle, looks like the figure on the right.



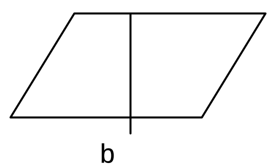
What is the figure on the right? A parallelogram. What is its area? The same as it was before you moved the triangular piece. So, what is the formula for the area of a parallelogram? A=bh

Summary

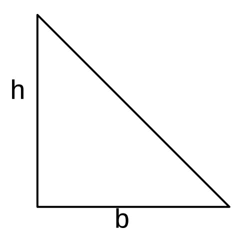
Rectangle A=bh



Parallelogram A=bh

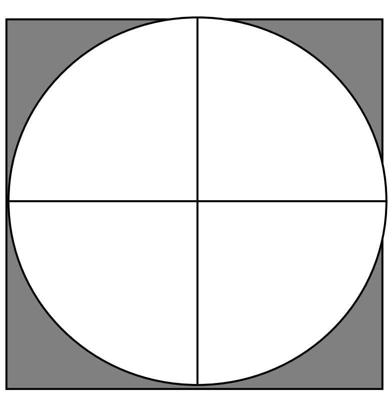


Triangle $A=\frac{1}{2}bh$



What about the area of a circle: A circle is "two-dimensional" also and you probably remember the formula for its area, $A=\Pi r^2$. Is it two- dimensional? Only one dimension, r, the radius, is given; but, notice this re-writing: $A=\Pi r\ r$ where the dimension r is used twice. r is multiplied by r just as b is multiplied by h in other area formulas.

You may skip the next paragraph if you wish. It's just a demonstration of the appropriateness in the formula of Π, which we may approximate, $\Pi=3.14$.

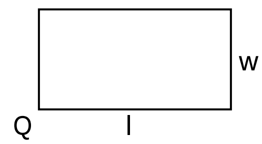


Consider a square with an inscribed circle. Divide the large square into 4 equal smaller squares. Notice that the length of the side of a small square is r, the radius of the inscribed circle. What is the area of one small square? r r or r^2. What is, therefore, the area of the large square? $4r^2$. The corners of the large square outside the circle are shaded to call attention to the fact that the area of the circle is smaller than the area of the large square. The area of the large square is $4r^2$; the area of the circle is 3.14 r^2. This seems reasonable.

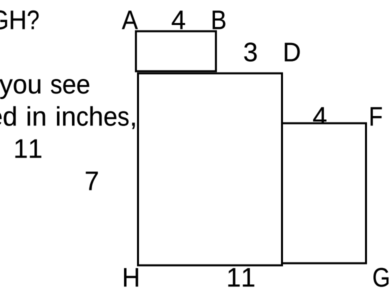
(Notice that we just slipped the area of a square formula $A=r^2$ into the discussion. After all, a square is a rectangle with A=bh. But in our example both the base and height were r so $A=r^2$ was OK. If we called the length of the side of the square s we would say $A=s^2$.

Perimeter

Perimeter is the measurement around something. The perimeter of a rectangle of length l and width w could simply be found by adding l+w+l+w. Just let your finger start at Q and "walk around" the figure. Of course, Perimeter, P=l+w+l+w could be written and calculated more efficiently by writing P=2l+2w. A little algebra would give yet another form. Factor out the 2 in 2l+2w and the formula becomes P=2(l+w).
Since you know that perimeter means the measure around, You don't really need a formula; just make sure you add up all the lengths that make up the outside of the figure.



Example What is the perimeter of the figure ABCDEFGH?



If you start at A and add 4+2+3+2+4+7+11+11=44, you see that the perimeter is 44 units. If it represents a figure measured in inches, the perimeter is 44 inches.

The "perimeter" of a circle is called the circumference and the formula for circumference is $C=2\Pi r$ or since the diameter, d = 2r, $C=\Pi d$ and that's just a fact. The way to show it is a fact is to measure lots of circles (their circumference and diameter) and show that the circumference divided by the diameter equals approximately 3.14 or Π. In symbols, $\frac{C}{d} = \Pi$.

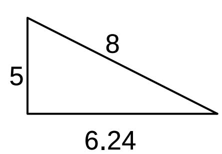
Certainly, it is efficient to have a list of formulas to find area and perimeter but if the formulas are just bunches of letters that don't have meaning it is much too easy to forget them or mix them up. The goal of this paper is to convince you that you can use your common sense and experience to recover forgotten formulas. Or, perhaps it's to show you that you can often find area or perimeter without using a formula.

Practice

1. To estimate how much fertilizer you need to use on a garden would you find the area or the perimeter of the garden?
2. To figure how much fencing you need for your garden would you find the area or the perimeter of the garden?

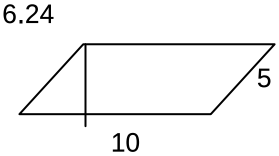
For exercises 3-6 find (a) the area and (b) the perimeter.

3. A rectangle, length = 3 feet; width = 1 7/8 feet
4. A rectangle, length = 17 in.; width = 11 in.
5. The triangle shown



(measured in inches)

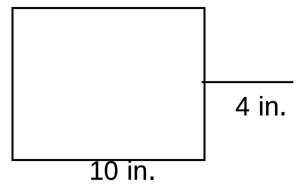
6. the parallelogram shown



(measured in centimeters)

Find (a) the area and (b) the circumference of the circles with dimensions given in exercises 7-9.

7. r = 3 in.
8. d = 11 ft
9. r = 4.5 cm
10. Find (a) the area and (b) the perimeter of the figure shown.



ANSWERS

1.	area		6.	(a) 40 sq. cm	(b) 30 cm
2.	perimeter		7.	(a) 28.27 sq. in.	(b) 18.85 in.
3.	(a) 5 5/8 sq. ft	(b) 9 ¾ ft	8.	(a) 95.03 sq. ft	(b) 34.56 ft
4.	(a) 187 sq. in.	(b) 56 in.	9.	(a) 63.62 sq. cm	(b) 28.27 cm
5.	(a) 15.6 sq. in.	(b) 19.24 in.	10.	(a) 105.13 sq. in.	(b) 40.57 in.